WHAT SHOULD COMPRISE THE SUBJECT MATTER OF THE ARITHMETIC IN THE ELEMENTARY SCHOOLS AND IN WHAT GRADES SHOULD FORMAL ARITHMETIC BE TAUGHT?

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The Lord, the school and society are responsible for the type of individuals that gets into the high schools, and after the Lord and society have done all that we can expect them to do for some time to come, there is left a much larger problem than simply to find the G.C.D. or the L.C.M. The child is an active thinking individual, if we do not suppress his activity and mechanize his thinking and convert him into a jumping-jack which responds only as the teacher pulls the strings and then apparently in a way that suggests need of lubrication. I have nothing but pity for the child who is allowed to think only through the ruts made by the juggernaut of mechanical teaching. Formal mental discipline, as interpreted by the Gradgrind martinet with its memoriter and rule-stuffing accompaniment, has been the fetish which has blocked the road for the development of childhood in mathematics for a long time.

It is no wonder that teachers of high schools say that the pupils they get are unable to take the initiative in any new type of reasoning and are unable to generalize and get the essence out of any considerable body of data. And these high schools get only the cream of the *academic-minded* pupils, whereas the grades are dealing with all types of pupils during the compulsory attendance period. The human weeds found in the grades are apt to interfere with the development of the type destined to reach the highest intellectual fruitage unless wise provision is made for their early segregation.

In this paper I shall emphasize aim and method rather than matter, because through aim and method, which will determine the matter and which will recognize the type of individual being dealt with, must come our salvation and our future better practice and results.

Elementary school arithmetic work divides into two general divisions in conformity to the psychological or natural division of the grades from the basis of a child's development, into the primary four years, and the grammar four years.

Formal work, as I interpret it in the topic given me for this paper, really begins with the grammar grades, the second four years. Formal work to me means logical step reasoning on problems (not process work or example solution). I would thus make formal work begin at the time that pre-solution statements begin to be emphasized. Mechanical solutions may be formal as to the machinery of production of answers, but they involve no reasoning on the part of the pupil, or so little consecutive logical thought that they are purely process imitations. The pupil acquires process forms through imitative doing from the very first, but reasoning is not harnessed to the performance until years later. We must always be mindful of the fact that one does not gain strength or power through the activity or exertions of another. We must also recognize the fact that children cannot assimilate the conclusions or rules worked out by any text-book-maker simply through memorizing the same and using them in a routine mechanical way to solve problems. We must not allow teachers or text-books to do all the organized thinking for the pupils and to present them with conclusions or rules to pigeon-hole in their memories. A pupil's reasoning must act upon the raw material of problems to make the why of doing clear. A rule, as well as a definition, is an abomination where it is the beginning rather than the conclusion of pupils' personal work and effort. To make myself clear perhaps this last statement needs explanation. Formulas like $2\pi R$ and πR^2, borrowing and carrying in subtraction and addition, inverting the fractional divisor to simplify work by multiplication, cancellation, etc., are simply taken as process data by the child, and reasons for their use are reserved until the pupil is ready to understand without a serious waste of time. A child of the fourth grade can gain skill and fix the operation or process of dividing one fraction by another, but it is unpedagogical to attempt to force the why upon him until one or two years later. To all of us some things clear up long after we have passed the performance stage. The child may learn a process through doing long before he is able to understand the whys for every

step, but his mental machinery should not be clogged by the meaningless verbiage of memorized rules.

The *primary work* must have as its basic trend conformity to the natural or physiological growth or development of the child, so that it will make the most of the keen memory of this period in fixing data and tables which must be rooted deep and firmly, and so that it will not attempt to force logical reasoning at a time when it will but blunt and stunt development.

The *grammar work* must be responsive and responsible to the every-day practical life demands, adjusted, to some extent, to fit the community experiences of the child.

In *primary work* the child occupies the center of the stage and all lines of effort converge to him rather than to the subject.

In *grammar grade work* the subject shares the arena and the method takes on large importance. As the express train is superior to the old Conestoga coach for transportation, so is one method, chiefly because of the aim back of it, superior to another in training children to initiative thinking.

The *primary work* must make real and clear the fundamental number relations chiefly with the play motive as the medium.

The *grammar work* must perfect skill in fundamental operations and establish processes for practical solution of all needed forms of problems. In this the preparation for high school is only a by-product of the preparation for life, not the aim of grade work.

Both *primary and grammar grades* give children power through much practice in getting their own data for problems, and in making problems, in order that they may see the conditions in their right relations in individual problems when taken from books.

Both *primary and grammar* grades give much practice in doing problems in distinction from simply working problems, in order to properly motivate the work, make it real, and make it function in commonsense results. For example, we want problems such as: Find the number of gallons of water that this tank or that tank will hold. Find the number of pupils' desks that can be legally placed in this room under the requirement of 200 cu. ft. of air per pupil. Get the proceeds of this note or of that note which will be put into the hands of pupils without further comment. Find the cost of reflooring this

room, or any other room specified, with first quality white maple, purchased of a definite dealer. This doing of problems in which pupils must take the initiative in ascertaining data as they would have to do in real life brings a touch of reality and interest into the arithmetic work that is absolutely vital and which adds tone and quality to the thinking and to the results. All right reasoning must come from a comprehensive view of the facts involved. We cannot always confine pupils to problems touching their limited experiences, but we can bring with the new problems new experiences if pupils find their own data. We cannot educate children against their will, and they have little will to acquire and little desire to work until they have fixed a definite motive as a result of interest. We must not, however, fail to note that problems which may be real and concrete to adults may be very unreal, foreign and out of time for pupils of a particular grade. Please mark that doing problems is a very different thing from working problems and develops an entirely different fiber of reasoning; the doing of problems develops power for initiative thought because it demands initiative thinking.

In the *primary grades* the forms of practical expression and solution including a knowledge of signs and terms are fixed. Solutions should be in direct, short-cut, business forms. It is foolish to express a series of numbers with plus signs between them when these numbers must be rewritten in columns for solution. All algorisms should be practical and economical of effort, at least in their habitual form. There is a field for someone to work out uniform algorisms that shall satisfy first psychological, and, second, service needs.

In the *grammar grades* the complete statement or expression in equation form of the entire problem before any work is done requires the pupil to think through the problem and note conditions fully, before there is any juggling with figures. This complete statement of most problems develops a more independent, rational type of thought power and very materially reduces, through cancellation, etc., the actual work necessary. The complete statement of problems as a method is vital also in that it overcomes the thinking by driblets which requires six dozen questions from the teacher to unloosen a conclusion that should spring from one or two definite questions, provided that

the pupil has sized up all the facts of the problem in their relations to each other and particularly to what is required. The complete statement trains pupils to turn their search lights on the conditions of a problem as an individual reality and think at it until they think through it and rivet a conclusion. It prevents the thinking in grooves as the adding machine works. Instead of producing pattern thinking or doing as working from rules and parrot explanations does, the complete statement produces constructive thinking which means independence in handling mathematical data. It is one means to get the pupils to think on their own hook and to think determinedly and not by starts and spasms.

The complete statement of arithmetical problems is the one most important factor in arithmetic for preparing pupils for algebra and geometry work. It overcomes the most serious cause of mistakes or errors by demanding an understanding of the conditions and language of a problem before trying a hit or miss solution. It frustrates any desire of teachers to waste time and distract attention of pupils by written step explanations during the time of solving.

In both *primary and grammar grades* the problems and work must enlist the pupil's real wish to do in the same sense that a game of ball gives him motive for self-activity. The character of the problems worked, particularly in the primary grades, needs radical change to meet the psychological rather than the logical or formal standard, otherwise it lacks the proper appeal to the child.

In both *primary and grammar grades* the natural element of competition comes in to help secure accuracy and speed. There needs to be exercises with time limits and time records as in a 100-yard dash. We need in teaching to make use of those forces which naturally impel pupils to self-exertion in their free hours; we need frequently intensity of mental attitude and attention; we need more active and less passive attention.

In both *primary and grammar grades* there should be close correlation with recreative and constructive exercises.

In both *primary and grammar grades* the home work should comprise exercises for skill and drill, but never new work not thoroughly understood. Home work should also aim to eliminate individual deficiencies in order to bring pupils into right

working relations with their fellows. It is archaic to expect every pupil of a class to do the same home work irrespective of their personal needs. Explanations, as well as attention, need to be focused upon difficulties.

If my hands were not tied, largely by tradition, I would put into operation a course in arithmetic, enlarging upon the fundamental idea that can be observed to-day in the city of Gary, Indiana, in which the data of arithmetic, the terms and tools of solution, and the processes of operation are acquired through the self-activity of actually doing real problems. As an example; I would require the eighth grade or finishing class to go into the town, select a house that they would wish to build, take their own measurements; stake out foundations on the school lot; compute the cost of building cellar and walls of house; get actual prices on materials; draw their own rough scale plans; floor and roof, plaster and paper the house; construct the chimneys, fire places, etc., etc. All of these things pupils would do from their own measurements and data. They would ascertain from material men the actual cost of materials and eventually would arrive at a definite cost for construction of a house of the type selected and would compare this cost with the actual cost of erection, checking their errors in estimates. I would have pupils borrow money on notes and through building loans; take out a mortgage on the property; have it insured; make out tax bills, etc., and through these operations get a review of all the phases and operations of arithmetic. Through such real work the development of the processes and the opening up of the subject comes by actually doing, which is the vital point in arithmetic preparation.

In manual training we have two types of teaching. In one we see pupils placed in rows on benches with notebooks and pencils and the teacher standing before them lecturing and dictating in reference to the parts and uses of chisels, planes, saws, hammers., etc., giving them definitions and rules of construction. In the other type of school we see the pupils determining upon something practical to make and learning all they need to know about the use and care of chisels, planes, saws, hammers, etc., through actually using these tools under the watchful eye and direction of the teacher. These pupils in the latter type of

school are doing something which has motivated the child's energy and his interest.

In arithmetic we see the same two types of teaching. In one, a class is riveted to recitation benches reciting on rules and definitions, working book problems that rarely hitch to the child's interest, getting rather mechanical, formal and indifferent results. In the other type of school we see pupils turning their thought power upon the actual conditions of real problems which they wish to solve, going into the fields, into the workshop, into the town, or wherever it may be, to get their data in the natural way, coming together with enthusiasm and interest in accomplishing something real, and eventually arriving at operations and methods of procedure and such definitions as may seem to be needed. I need not comment upon which type of work gives the highest type of result as to tone and quality.

My experience in Altoona, Pa., in connection with a group of retarded or slow-developing grammar grade pupils that were given an opportunity to go into the school machine shops and take a course in practical shop mathematics, leads me to feel that if we get our pupils away from book work a great deal more than we do and bring them to real problems where they must do things and find some of their own data that we will accomplish an entirely different type and quality of thinking and will develop some originality and initiative in our pupils and will overcome most of the criticisms that are being heaped upon grade schools and will save to our high schools and higher institutions many that now become discouraged and fall by the wayside.

But as I am hampered by tradition and by a board of education who are hampered by tradition and customs of the past and as school men in general are so hampered, it is necessary for us to lay out a course in mathematics with general boundary lines and fences that do not shatter too rudely the ideas of our forefathers. We must, however, put in entry wedges as frequently as possible for the new type of work and particularly psychological methods and aims and with these thoughts I submit virtually the outline of work that I have arrived at after a lifetime of experience as superintendent and study of school needs.

First Grade.

The first grade work aims to give the child a clear and real idea of the number relations in the way that he naturally acquires facts, through pleasurable activities, games, relaxation and occupation exercises. These number relations are confined to the number as a whole and to the plus, minus, times, division and fractional relationships up to and including 10.

Baby talk is tabooed. Whenever a term is needed the term, as it will be used later, is given; thus the child from the first uses plus, minus and all the signs of operation without conscious drill on these terms. All terms of method, process and operation are used incidentally as a wise parent would use the word chair in implanting in the baby's mind the idea. He would say: Sit in the chair. Stand by the chair. Put the book in the chair, etc. The child learns the word chair through its relationships with his life activities. Nothing is more deadening and dwarfing to the thinking power of the first or second grade child than mechanical, formal arithmetic drill work.

To illustrate what I mean by games to establish number relations I give one of a great variety of exercises used. Some child is chosen as a leader to play wild horse; he runs down the aisle and taps five children who come to the front of the room prancing up and down. The leader announces that he captured five wild horses which he put in a pen, but three of them have jumped the fence and ran away. The teacher places on the blackboard $5 - 3$, and when the pupil completes his statement by saying that he has 2 horses left, she puts down the 2 after the equality sign on the board and reads the statement 5 less 3 are 2. Later on the pupil will write the whole expression himself which merely represents what he has done just as the sentence, John hit the ball, represents something that was done right in the school room before the teacher wrote the sentence on the blackboard. The teacher asks the pupils to listen and repeat what she does and then she raps 5 times and then says minus and then raps three times. The pupils do the same, then they run to the blackboard and write $5 - 3 = 2$. The purpose of this is to fix the idea of subtraction, not as a bookish fact, but as a reality and as something that the child uses and can use. The data and figures that a child must later have are

simply thrown in as incidentals and the child comes to know that these forms are used just as words are used to express certain relations of things that he deals with. The resourceful teacher will bring in many things that are of vital interest to her particular children and will teach the addition relation, the multiplication relation, the subtraction relation, the division relation, the partition or fractional relation entirely through the games and exercises that will furnish activity, physical relaxation and abundant pleasure and enjoyment for her pupils.

I have no use for the "Nancy" exercises in arithmetic, such as abstract 2 and 1, 2 and 2, 2 and 3, 2 and 4 in rotation around a class during which pupils are glued to their seats. I want some virility to the work from the very first minute. Instead of making school life hateful, arithmetic is one of the very best subjects to give the pupils the freedom of the class room and school and take away that timidity and restraint that is such a barrier between home life and the first year of school life.

I have still less use for those dry bones of arithmetic so common in classes presided over by the formal, methodical drill master; I want some flesh and blood, some life and activity; I want some emphasis placed upon the psychological and natural treatment of the subject rather than upon the logical treatment considered from the adult standpoint. The first grade is the most important grade in the school system.

Second Grade. (Method most Important.)

Game work of the first grade continued.

Picturing and expressing on blackboard combinations that form numbers to 10 as occupation work. (Domino work.)

Counting by 2's, 3's, 4's, 5's, 6's.

Master times tables, including 6's.

Dividing by 2 up to numbers of 6 times table.

Counting by 2's, 3's, 4's, starting with 1, 2, 3, 4, 5, and 6.

Fix addition facts by practice in writing and adding three numbers of 3 or 4 figures each.

Subtraction should be treated as a correlative process of addition, so that when adding $3+5=8$ pupils will see $8-5=3$. (Subtracting by adding eliminates one set of tables.)

$\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ of numbers to 10 which give integers as answers.

Trade questions involving change up to 50 cents as subtraction drill.

Inch, $\frac{1}{2}$ ft., 1 ft. and use of ruler in paper construction work and in measuring objects, and blackboard work in drawing to measure.

Multiplying, and testing results by addition.

Roman numbers to 20.

Fix habit of checking and testing answers.

Sufficient doing problems to make real and clear the data to be memorized.

Pupils make problems from abstract statements such as $4+3=$; also problems based upon games or imaginary conditions. (Conversion of examples into problems.)

Third Grade.

Continue occasionally game work so that children will not come to look upon number relations as a bookish matter.

Make automatic all the times tables including the 10's as well as all addition and (subtraction) and (division) facts.

Dry, liquid and linear measures with simple changes of unit of measure. Practical problems.

Reading and writing Hindu numbers. Pointing off as write. Roman numerals to the extent found in readers.

Addition of mixed numbers as

$$\begin{array}{r} 8\frac{2}{3} \\ +\,6\frac{1}{3} \\ \hline \end{array} \qquad \begin{array}{r} 9\frac{1}{3} \\ +\,14\frac{2}{3} \\ \hline \end{array}$$

Multiplication by easy mixed numbers.

Short division with divisor 10 or less. Picture problems, considering division as correlative process of multiplication.

Speed contests in multiplication and addition examples.

Practical review of and drill in all fundamental operations.

Emphasize comparison of values. Whole numbers and fractions with problems. Compare 2 with 4, compare 4 with 2, compare $\frac{1}{2}$ with 2, etc.

Time reading, calendar construction, table.

Practical surfaces constructed and computed—doing problems through construction work. Candy and glove boxes, mats, tiles.

From market reports find cost of articles used in home.

Bills, and work with money and decimal point in all the operations.

FOURTH GRADE.

This grade masters thoroughly the four fundamental operations with all signs and practical forms of solution.

It makes a thorough study of all fractional operations in problems of intermediate difficulty.

It masters tables and works easy problems in measures, weights, surfaces, contents, time.

Sight work with round numbers and easy parts of one dollar.

Multiplication and division of numbers ending in one or two ciphers.

Long division mastered with quotient always placed above dividend as preparation for decimal work.

Squares of easy numbers with pictures of same.

Master reading and writing numbers to billions.

Bank accounts as problems in addition and subtraction.

Add and subtract mixed numbers. Multiply whole by mixed numbers.

Comparison of values in solution of problems.

Bills from buying schedules.

Reduction of practical denominate number terms to higher and lower terms.

Pupils make problems; solve problems in which they secure their own data; as, walks, floors, window panes, bins, etc.; practical book problems, emphasizing short processes. Check and test results.

Complete statements of many problems without working.

Competitive speed drills.

Addition and subtraction of mixed numbers involving equalizing of denominators of fractions.

Fractional parts of numbers representing money, denominate expressions, concrete and abstract quantities.

Cubic contents—secure data; build in construction work; solve rational problems.

(*To be Continued.*)